

Amplitude, Intensity and Loudness

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Under normal circumstances, sound takes the form of variation in the pressure of the air about the average ambient air pressure. The most direct measurement of sound therefore is the instantaneous excess pressure, measured in standard units of pressure. Pressure has the dimensions of force per unit area. In the cgs system, it therefore has units of dynes per square centimeter (*dynes cm⁻²*). One dyne is defined as the force necessary to accelerate one gram of matter at a rate of one centimeter per second per second. In terms of fundamental units, a dyne therefore has dimensions of *gm cm s⁻²*.

In the MKS system, pressure is measured in terms of Newtons (N) per square meter, where one $1 N = 10^5 \text{ dyne}$. In the MKS system a special name, the *Pascal*, is given to the unit of pressure. $1 Pa = 1 N m^{-2} = 10^5 \text{ dynes } m^{-2} = 10 \text{ dynes } cm^{-2}$.

There are two other systems for measuring pressure. An old-fashioned system still occasionally encountered is in centimeters of water or millimeters of mercury. These refer to the height of a column of the specified liquid. You can convert from centimeters of water to millimeters of mercury by dividing by 1.36 since the density of mercury is 13.6 times that of water. Another system, which refers to the pressure of the atmosphere, is used mainly by scuba divers and submariners. A variant of this system, using the finer subdivisions of millibars and microbars, is often used by meteorologists.

Here is a chart giving the equivalent of 1 *dyne cm⁻²* in other units. The last column gives the multiplier to use in the inverse direction, that is, for converting from the other unit to dynes per square centimetre.

(1) Units of Pressure Measurement

| | | |
|------------------------|--|---------------------|
| 1.000×10^{-1} | Pa (Pascals) | 1.000×10^1 |
| 1.020×10^{-3} | cm H ₂ O (centimeters of water) | 9.807×10^2 |
| 7.500×10^{-4} | mm Hg (millimeters of mercury) | 1.334×10^3 |
| 9.869×10^{-7} | atm (atmospheres) | 1.013×10^6 |
| 9.869×10^{-7} | bars | 1.013×10^6 |
| 9.869×10^{-4} | millibars | 1.013×10^3 |
| 9.869×10^{-1} | microbar | 1.013×10^0 |

Atmospheric pressure, defined as the average pressure at sea level on the equator, is approximately 1.01325×10^6 dynes per square centimeter. This is equivalent to 1033 cm H₂O or 760 mm Hg.¹

¹ Pressure decreases as altitude increases according to the formula $P = 1.0 e^{-(h/7000)}$ where P is in atmospheres and h is in meters. Pressure increases with depth in water according to the

The instantaneous amplitude of a sound is therefore the instantaneous excess pressure measured in *dynes cm⁻²*. This should not be confused with the overall amplitude of the sound.

In the case of some familiar functions, the word *amplitude* is the name of the parameter that determines the scaling of the function. For example, the general form of the sine function is:

$$(2) \quad g(t) = A \sin(2\pi ft + \phi)$$

Here f is the frequency in cycles per unit time, ϕ is the phase, in radians, and A is the amplitude. In this case, the value of the function varies over time, and the overall amplitude of the function is actually reached only at the peaks.

When we do not have such a complete mathematical description of a function with a single “size” parameter, or where we want to talk about the overall amplitude of a region of a function, we need some kind of average over time. The average usually used is the *Root-Mean-Square* (RMS) amplitude of the function. The RMS amplitude of $f(x)$ over the region $[a, b]$ is defined in the continuous case as:

$$(3) \quad \sqrt{\frac{\int_a^b f^2(x) dx}{b - a}}$$

In the case of digital data, this takes the discrete form:

$$(4) \quad \sqrt{\frac{\sum_{i=j}^k f^2(i)}{k - j + 1}}$$

What this means is that we take each value of the function within the region of interest, square it, add up the squared values for each of the points within the region, divide by the number of points to obtain the mean, and then take the square root of this sum. Since the individual values have dimensions of *dynes cm⁻²*, the sum has dimensions of *dynes² cm⁻⁴*, and its square root again has dimensions of *dynes cm⁻²*. One reason for squaring the individual values is that this prevents the negative values from cancelling out the positive ones.²

It is important to keep clear several physical notions. *Intensity* is defined as the average rate of flow of energy per unit time per unit area. *Power*, defined as the rate of flow of energy per unit time, is the integral of intensity over the area through

formula $P = 1.0 + \frac{h}{10}$. The negative exponential in the first formula reflects the fact that the density of the atmosphere decreases with altitude. In contrast, the variation in the density of water with depth is small so a linear formula gives a close approximation.

² Look at a graph of a sine wave. If you take an ordinary average over an integral number of periods, the result will be zero because the positive and negative portions cancel each other out.

which the sound is flowing. Power therefore has dimensions of energy per unit of time. It is usually measured in *watts*. Intensity has dimensions of power per unit area, so it is usually measured in units of watts per square centimeter.

The intensity of sound is given by the following equation, where p is the effective pressure in dynes per square centimeter, ρ_0 is the density of the medium through which the sound is travelling in grams per cubic centimeter, and c is the speed of sound in the medium, in centimeters per second.

$$(5) \quad I = \frac{p^2}{\rho_0 c}$$

Note in particular that intensity is proportional to the square of the pressure. We can rewrite the above equation as:

$$(6) \quad I = kp^2$$

where the constant of proportionality k is:

$$(7) \quad \frac{1}{\rho_0 c}$$

The loudness of sounds is usually not described in the ordinary pressure units of dynes/cm². Rather, loudness is described in terms of *decibels*. Decibels are basically a measure of a power ratio, defined as follows:

$$(8) \quad dB = 10 \log_{10}\left(\frac{W_1}{W_2}\right)$$

where W_1 and W_2 represent quantities with dimensions of power, the rate at which work is performed, measured in watts (joules per second). The decibel is just 1/10 of a Bel, a unit named after Alexander Graham Bell, the inventor of the telephone. Thus, leaving aside the multiplier of 10 that converts Bels to decibels, decibels are just the logarithm of the ratio of the powers of the two signals compared.

$\log_{10}(x)$ is the logarithm of x to the base 10, or *common* logarithm.³

If we know that two signals differ by a given number of decibels, we can invert formula (8) to obtain the ratio of their powers:

$$(9) \quad R = 10^{dB/10}$$

³ If natural logarithms are used instead, the resulting measure is known as the *neper* instead of the *Bel*. I have never encountered the neper in actual use.

When we say that one signal is 20 dB above another, we are saying that the ratio of their powers is $10^{20/10} = 10^2 = 100$. If one signal is 30 dB above another, the ratio of powers is $10^{30/10} = 10^3 = 1000$. If one signal has a power of 1 watt and the other signal is 30 dB above it, the second signal has a power of 1000 watts.

Decibels used to express power ratios in this way occur in two main situations in speech. Sometimes they are used to express the *signal-to-noise-ratio* (often abbreviated SNR). This is the ratio of the signal, typically the speech, to the noise. If we are told that a recording has an SNR of 45 dB, that means that the power of the signal is $10^{45/10} = 10^{4.5} = 31622.8$ times the power of the noise.

The second situation in which decibels are used to express ratios is in describing filters. A *passive* filter does not contribute any energy to the signal; it just transmits some of the energy in the input. A linear passive filter transmits a certain proportion of the energy at each frequency. Since we often don't care very much about how much the filter reduces the overall volume of the signal, but are interested in the way in which it shapes the spectrum of the signal, filter characteristics are often plotted in decibels with respect to the maximum. In this case, the proportion of the power transmitted at each frequency is being compared to the proportion of energy passed at the frequency at which the most energy is transmitted.

In speech decibels are also used to measure the absolute amplitude of sounds. This may seem confusing, as decibels by definition represent power *ratios*, not absolute levels. The key is that when decibels are used to measure absolute levels, the measured signal is implicitly compared with a reference signal. This reference signal is chosen to be approximately the faintest audible sound, so that audible sounds will generally have positive decibel values. Since the sensitivity of the ear varies with frequency, the reference level is arbitrarily set to the faintest sound audible at 1000 Hz.

When we are given the intensity level of a signal, we need merely use formula (1) with W_1 replaced by I , the intensity of the signal, and W_2 replaced by I_{ref} , the reference intensity. This decibel measure is denoted dB_{IL} , where *IL* stands for “Intensity Level”.

$$(10) \quad dB_{IL} = 10 \log_{10}\left(\frac{I}{I_{ref}}\right)$$

$$(11) \quad I_{ref} = 10^{-16} \text{ watts/cm}^2$$

dB_{IL} are not used often in speech research. They are more often found in work on architectural acoustics and noise control. In speech research the amplitude measure available to us is usually sound pressure level, since most microphones produce a voltage that is proportional to pressure, or, strictly speaking, to the deviation of the pressure from the average ambient air pressure. When the available measurement is pressure, the decibel measure used is dB_{SPL} , where *SPL* stands for “Sound Pressure

Level”. dB_{SPL} are defined in (12). Here again, the reference level, given in (13), is approximately the minimum audible sound for a tone of 1000 Hz.⁴

$$(12) \quad dB_{SPL} = 20 \log_{10}\left(\frac{P}{P_{ref}}\right)$$

$$(13) \quad P_{ref} = 2 * 10^{-4} \text{ dynes/cm}^2$$

Except for the use of a pressure measure and a different reference level, formula (12) differs from formula (10) only in that the multiplier is 20 rather than 10. This is because decibels are always a measure of a power ratio, even when the measure we wish to use is pressure, and power is proportional to the square of pressure. Let us take formula (10) and substitute for W_1 and W_2 equivalent expressions in terms of pressure. For our present purpose, the value of k does not matter, but we may recall that we in fact know that $k = \frac{1}{\rho_0 c}$.

$$(14) \quad dB = 10 \log_{10}\left(\frac{kP_1^2}{kP_2^2}\right)$$

The two k s cancel, so we may rewrite this as (15).

$$(15) \quad dB = 10 \log_{10}\left(\frac{P_1}{P_2}\right)^2$$

Now, using the fact that the logarithm of the square of a quantity is twice the logarithm of the quantity, we may rewrite (15) as (16).

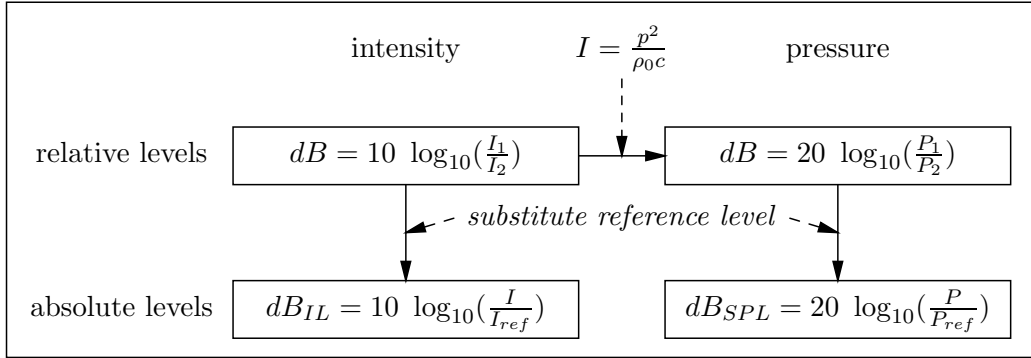
$$(16) \quad dB = 2 * 10 \log_{10}\left(\frac{P_1}{P_2}\right) = 20 \log_{10}\left(\frac{P_1}{P_2}\right)$$

If we replace P_1 by P and P_2 by P_{ref} , we obtain (12).

The logic of the relationships is illustrated by the following diagram.

⁴ There is still another variant of decibel measurement, known as dB_{SL} for “Sensation Level”, in which the reference level varies with frequency, since the ear is more sensitive to some frequencies than to others.

(17) Relationships of Decibel Formulae



To give an idea of typical decibel levels, here are approximate values of a variety of sounds in dB_{SPL} .

(18) Loudness of Various Sounds

| Sound | dB_{SPL} | dynes/cm ² |
|-----------------------------------|------------|-----------------------|
| Threshold of hearing | 0 | 0.0002 |
| Rustling of leaves | 20 | 0.0020 |
| A typewriter | 40 | 0.0200 |
| Conversation | 60 | 0.2000 |
| Shouting | 80 | 2.0000 |
| Approaching subway train | 100 | 20.0000 |
| Jet airplane for person on runway | 120 | 200.0000 |
| Damage to ears | 140 | 2000.0000 |

Decibels are used for two reasons. First, the audible range of sound pressures and intensities is very large, as we can see from the values in the third column in (18). Using a logarithmic measure like decibels compresses this range and makes the numbers more manageable. Second, our perception of loudness is not linear — a sound with twice the sound pressure level of another is not perceived as twice as loud. Loudness perception follows a psychophysical law known as *Weber's Law*, according to which loudness increases with the logarithm of sound pressure. Decibel measures of amplitude therefore reflect more accurately perceived loudness than linear measures do.

Here are plots of dB_{SPL} as a function of the sound pressure level in dynes/cm². Because the range of pressure values is so large, we have broken it up into two plots.

