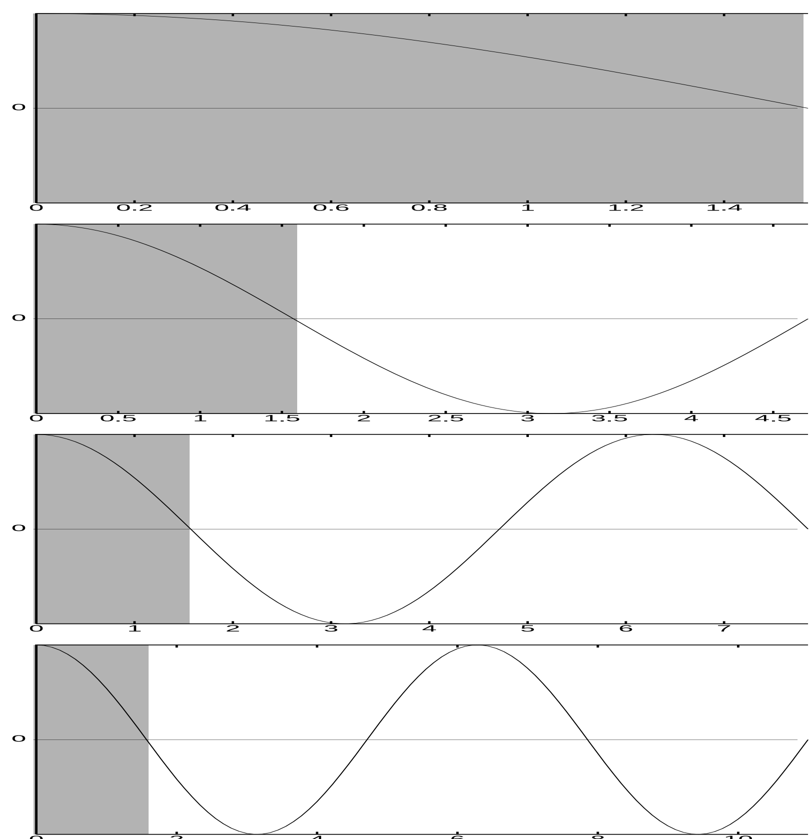
The Resonances of a Uniform Tube Closed at One End

We consider a uniform tube closed at one end and open at the other. This tube is a reasonable approximation to the vocal tract in a schwa-like configuration, that is, when there is no signficant narrowing anywhere and where the velar port is closed. The closed end of the tube represents the very high impedance at the glottis. The open end represents the very low impedance of the open mouth.

A wave will resonate in such a tube when the pressure maximum is at the closed end of the tube and the pressure minimum (in absolute terms, that is, the zero point, not the largest negative value) is at the open end. We can fit the first quarter of a cosine into a tube in such a fashion, as shown in the first diagram below. However, since the second and third quarters of a cosine descend from zero and return to zero, we can add the next two quarters of a cosine and still meet the requirement that the pressure maximum be at the closed end and the minimum be at the open end, as seen in the second diagram. Similarly, since the last quarter of a cosine followed by the first quarter of the next wavelength of a cosine ascend from zero and return to zero, we can also satisfy the requirement by adding this portion of a cosine. We can of course add a full wavelength of a cosine, as seen in the third diagram. In the fourth, we have added 1.5 wavelengths.



$L = \frac{\lambda}{4}$

$L = \frac{3\lambda}{4}$

$L = \frac{5\lambda}{4}$

$L = \frac{7\lambda}{4}$

Generally speaking, since each half-period, starting after the first quarter period, begins and ends at 0, we can insert any integral number of half periods and still have a pressure maximum at the closed end of the tube and a pressure minimum at the open end. In other words, odd numbers of quarter wavelengths fit exactly into the length of the tube. We therefore have

$$\frac{(2k-1)\lambda}{4} = L$$

where L is the length of the tube in centimeters.
We know that

$$C = f\lambda$$

where C is the speed of sound and f is the frequency.
Solving this for the wavelength λ we have

$$\lambda = \frac{C}{f}$$

Substituting this expression for λ, we have

$$\frac{(2k-1)C}{4f} = L$$

To solve this for f we first multiply both sides by f:

$$\frac{(2k-1)C}{4} = fL$$

Dividing both sides by L and reversing the order of the two sides of the equation, we obtain:

$$f = \frac{(2k-1)C}{4L}$$

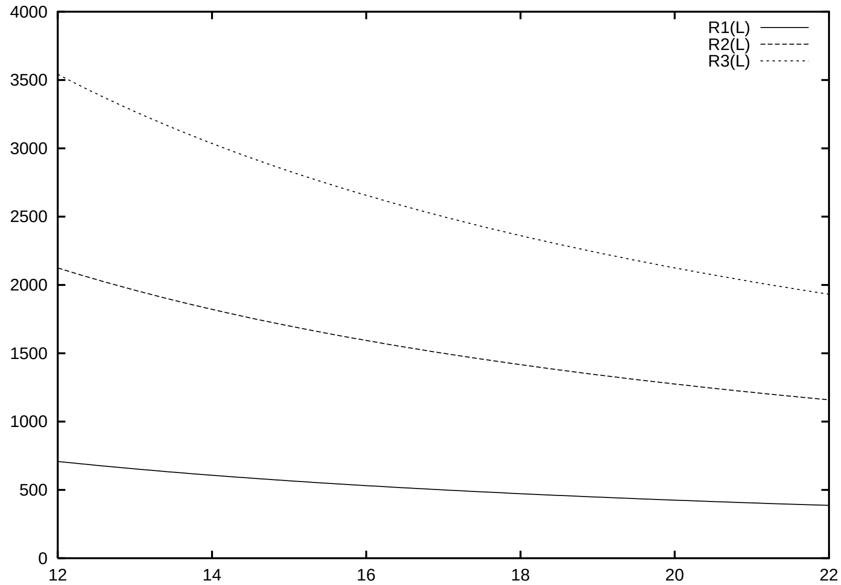
Since these are the resonant frequencies, we shall denote them as R_k, so our general expression for the resonant frequencies becomes:

$$R_k = \frac{(2k-1)C}{4L}$$

The speed of sound depends on the medium and the temperature. A reasonable value to use for our purposes is 340 m/s, which is equivalent to 34,000 cm/s. We may use 17cm as the length of a typical adult male vocal tract. With these values, we obtain:

$$\begin{aligned} R_1 &= 500Hz \\ R_2 &= 1500Hz \\ R_3 &= 2500Hz \end{aligned}$$

The dependance of the resonant frequencies on the length of the vocal tract is illustrated in the following graph. Notice how lengthening the vocal tract not only lowers the resonant frequencies but reduces the separation between them.



The First Three Resonances as a Function of Vocal Tract Length

It is important to remember that the resonances of the vowel tract are the frequencies at which it will transmit energy best, not necessarily the frequencies at which there will be the maximum energy in the output. The vocal tract is only serving as a filter, so the spectrum of its output depends not only on the vocal tract but on the spectrum of the sound sources that constitute the input to the filter. If the source energy is evenly distributed, the peaks in the output spectrum (that is, the formants) will be located at the resonances, but if there is little input energy at a resonant frequency, there will not be much in the output.