Separating Shape and Gain in Filter Characteristics

A filter can be thought of as performing two functions. One is modifying the *relative* proportions of energy transmitted at different frequencies. The other is modifying the total amount of energy transmitted.

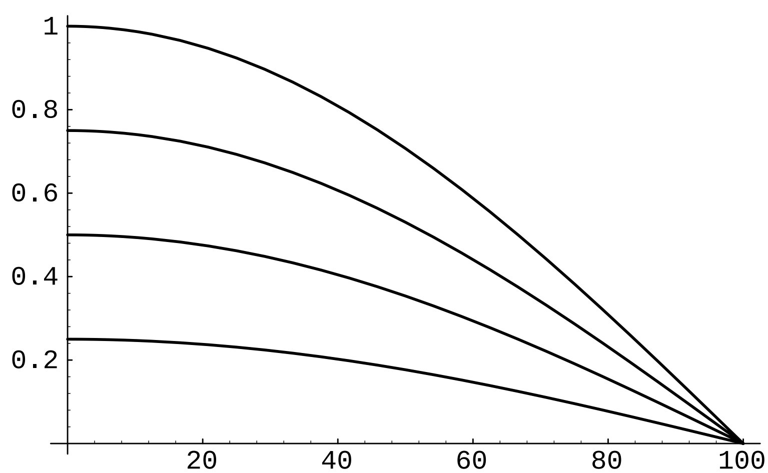
In phonetics we are usually dealing with filters that are *linear* and *passive*. A linear filter is one that does not move energy from one frequency to another. It may change the spectrum by transmitting a greater fraction of the energy at one frequency than at another, but it does not shift energy around. A passive filter is one that does not add any energy to its input. In other words, a passive filter can cut back on its input, or it can redistribute input energy to different frequencies, but it cannot contribute any energy of its own. A linear, passive filter then is one that can change the spectrum only by selectively reducing the energy transmitted at certain frequencies.

Filters that transmit energy at different frequencies in the same proportions are said to have the same shape, but possibly different gains. The volume control on a stereo amplifier is a pure gain control. It increases or decreases the total amount of energy output by the amplifier without affecting the mixture of frequencies. A bass or treble control, on the other hand, affects the mixture of frequencies.

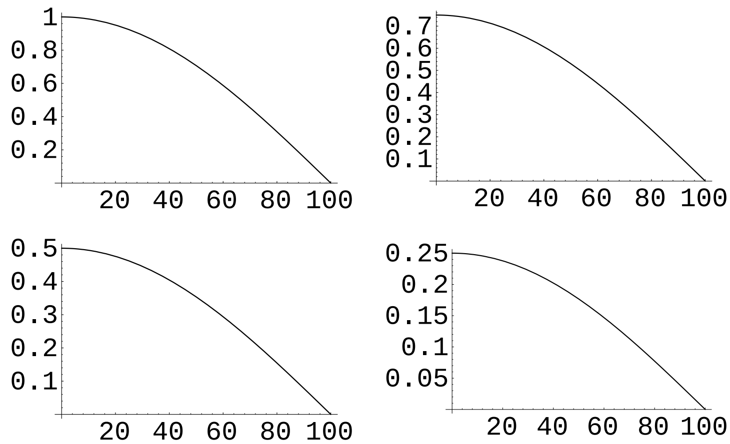
If two filters have the same shape but different gains, you can build a system that mimics one filter by adding an amplifier/attentuator to the other filter and setting its gain (volume control) to an appropriate level.

We often care only about a filter's shape, not its gain, so it is useful to have a way of representing filter characteristics that is independent of gain. Also, it is often convenient to express a filter characteristic in decibels since we can then add these decibels to the input amplitude at each frequency, also in decibels. The resulting output is, of course, only accurate up to a constant multiplier reflecting the unknown gain of the filter.

Consider a set of filters consisting of the first quarter period of a cosine of period 400, with amplitudes 1.0, 0.75, 0.5, and 0.25. These are plotted below:



If we scale the ordinate so that the maximum value is at the top, as in the four plots below, we see that the filter characteristics have the same form. That is because we can transform one into the other simply by multiplying by some constant (e.g. by 2.0 to transform the filter with amplitude 0.5 into the filter with amplitude 1.0).



If we plot the ratio of the proportion of energy transmitted at each frequency to the proportion transmitted at the maximum, we obtain the same effect on the plot as scaling the ordinate, but we lose the information about the overall amplitude. All four plots will have the form of the plot with amplitude 1.0 above. If we take the logarithm to the base 10 of this ratio and multiply by ten, we obtain the filter characteristic shown below, whose ordinate is in $dB\ re\ Max$, decibels with respect to the maximum.